ESG, Risk, and (tail) dependence

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Introduction

- The availability of non-financial data, including environmental, social, and governance (ESG) data has skyrocketed, and has gained great interest from academics and practitioners.
- ESG scores are based on several criteria given by a rating institution (Bhattacharya and Sharma, 2019). The rating institutions use quantitative and qualitative methods to assign an ESG score to a company (Berg and Lange, 2020).

EIKON Thomson Reuters ESG Scores

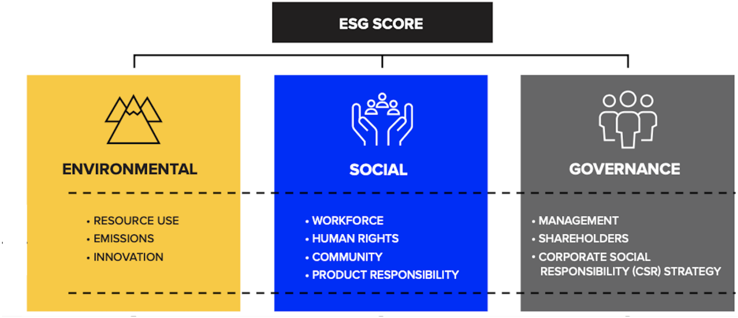


Figure 1: Aggregate ESG Scores

Previous Works

- It has been suggested that companies with better ESG scores enjoy better financial and market performance (Aboud and Diab, 2019).
- However, adding ESG criteria to an equity portfolio may not necessarily yield any additional returns (Breedt et al., 2019).
- No-consensus has yet been found on the link to corporate financial performance, as the results differ depending on the data used and the design of the study (Junkus and Berry, 2015; Shafer and Szado, 2018; Friede, 2019; Dorfleitner et al., 2020)

Motivation

- While more than 2000 empirical studies have been done analysing ESG scores and financial performance, little has been done to understand the dependence structure and associated risks.
- Regulatory authorities, such as the European Banking Authority (EBA), have acknowledged that ESG scores can contribute to risk.
- This research aims to question whether ESG score can allow to capture (tail) dependence and (tail) risk to a certain degree.

95% Value-at-Risk

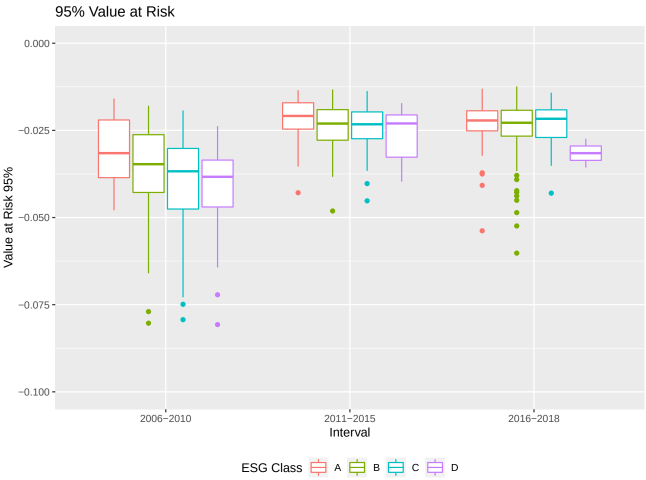


Figure 2: 95% VaR with ESG Grade A (75$<$ score $\leq$ 100), B (50 $<$ score $\leq$ 75), C (25 $<$ score $\leq$ 50), D (0 $<$ score $\leq$ 25)

Dependence Modelling

A copula C is a cumulative distribution function (cdf) with uniform marginals on the $[0,1]$.

Sklar's theorem (1959) states that if F is a continuous d-dimensional distribution function for $\mathbf{X} = (X_1, \ldots, X_d)^\top$ with a univariate cdf $F_p(x_p)$ of a continuous random variable X_p for $p = 1, \ldots, d$ with its realizations x_p, the joint distribution function F can be written as

$$F(x_1, \ldots, x_d) = C(F_1(x_1), \ldots, F_d(x_d)). \tag{1}$$

The corresponding density is

$$f(x_1, \ldots, x_d) = c(F_1(x_1), \ldots, F_d(x_d)) \cdot \prod_{p=1}^{d} f_p(x_p), \tag{2}$$

where C is some appropriate d-dimensional copula with copula density c.

Multivariate Copula Model

However, there are some drawbacks to multivariate copula models. These include the inflexibility in larger dimensions and the restriction of similar dependence structures between the variable pairs.

To overcome these issue and as we are interested in separate multivariate component modeling, we fit *vine copula* models instead.

Vine Copula Model

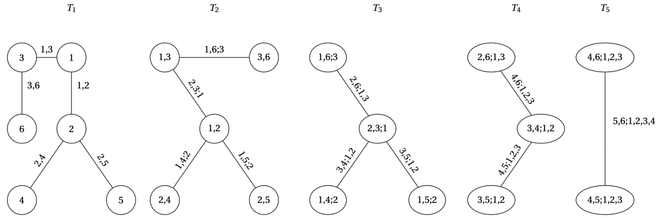


Figure 3: Example of a 6-dimensional regular vine with edge labels.

	Itau[1] Copulas							BB Copulas		
Properties	*t*	*F*	*N*	*C*	*J*	*G*	**I**	*BB1*	*BB7*	*BB8*
Positive Dependence	✓	✓	✓	✓	✓	✓	-	✓	✓	✓
Negative Dependence	✓	✓	✓	-	-	-	-	-	-	-
Tail Asymmetry	-	-	-	✓	✓	✓	-	✓	✓	✓
Lower Tail Dependence	✓	-	-	✓	-	-	-	✓	✓	-
Upper Tail Dependence	✓	-	-	-	✓	✓	-	✓	✓	✓

Table 1: Parametric copula families and their properties without rotations and reflections. Notation of copula families:t = Student's t, F = Frank, N = Gaussian, C = Clayton, J = Joe, G = Gumbel, **I** = Independence, BB1 = Clayton- Gumbel, BB7 = Joe-Clayton, BB8= Extended Joe

[1] Copula families for which the parameter estimation by Kendall's τ inversion is available without rotations

Data

- Daily log returns of 334 companies j (constituents the S&P 500)
- Time frame: $y = 2006 - 2018$
- Trading days: $t = 1, ..., 3271$
- Time periods q : 2006-2010, 2011-2015, 2016-2018
- Sectors: $S = 1, \ldots, 10$
- Yearly ESG Scores ($ESG^{S}_{j,y}$)
- S&P 500 Sector indices ($I^{S,q}_{t}$)
- S&P 500 market capitalization weights (by 1.01.2015) to compute the ESG class indices $\mathbf{I}^{S,q}_{t,k}$

Sectors

Sector (S)	Count (j)
Basic Materials	19
Consumer Cyclicals	50
Consumer Non-Cyclicals	31
Energy	16
Financials	49
Healthcare	35
Industrials	41
Real Estate	21
Technology	51
Utilities	23

Table 2: Number of Assets j per Sector (S)

ESG Scores - Trend

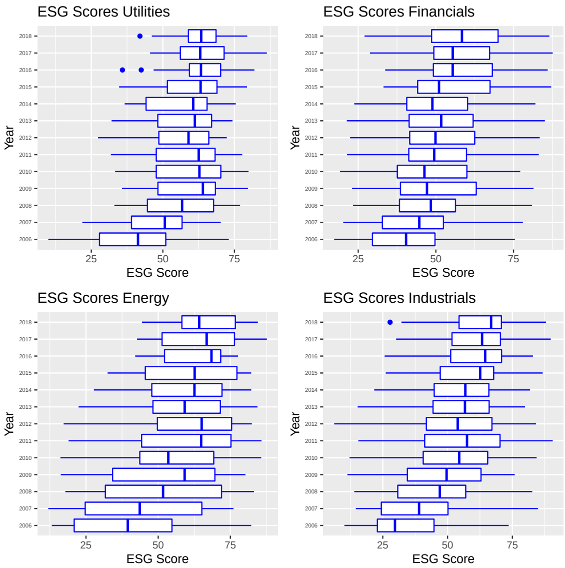


Figure 4: Boxplot ESG Scores

ESG Class Indices - Clustering

We cluster data companies according to their mean ESG score over the specific interval.

We have $\forall_{j,S,q}$:

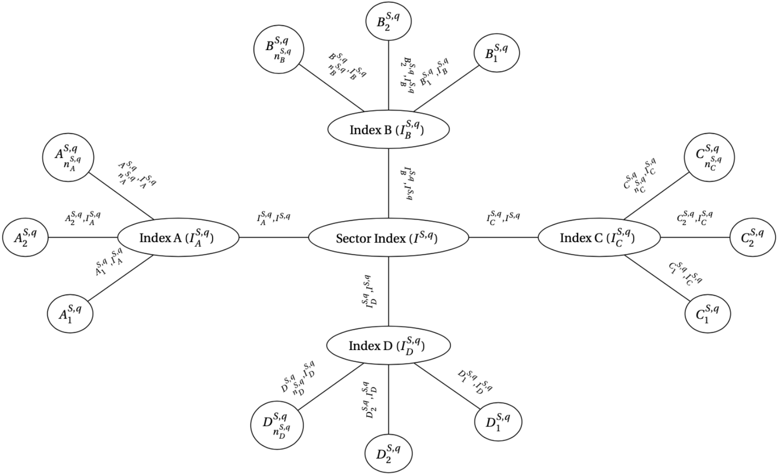
$$R_j^{S,q} = \begin{cases} A, & \text{if } \overline{ESG}_j^{S,q} \in [75, 100], \\ B, & \text{if } \overline{ESG}_j^{S,q} \in [50, 75), \\ C, & \text{if } \overline{ESG}_j^{S,q} \in [25, 50), \\ D, & \text{otherwise.} \end{cases} \tag{3}$$

This is necessary to compute our ESG class indices ($\mathbf{I}_{t,k}^{S,q}$)

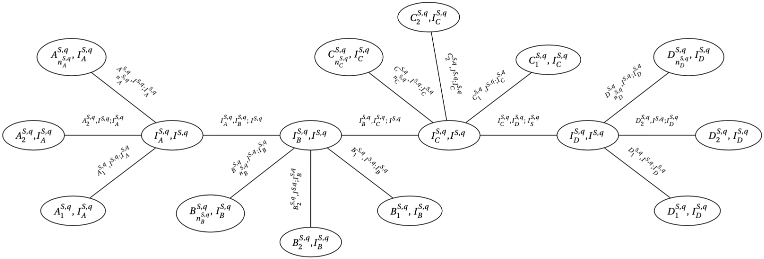
Two-Step Inference for Margins

- Financial data are strongly dependent on past values and not uniformly distributed on $[0, 1]^d$, therefore, a two-step inference for margins (IFM) approach is followed.
- We propose a parametric marginal model and estimate the margins first, we then use the estimated marginal distributions to transform the data on the copula scale by defining the pseudo-copula data.
- A GARCH(1,1) with Student t innovations is fitted to each financial return series, Sector index, and ESG Class Indices, allowing for time-varying volatility and volatility clustering.
- Using the cumulative distribution function of the standardized Student t distribution, we determine the pseudo-copula data using the probability integral transform (PIT).

R-vine Copula ESG Risk Model - Tree 1



R-vine Copula ESG Risk Model - Tree 2



ESG class dependence indicator $D_j^{S,q}(\tau)$

For each asset j with its ESG class k in period q within each sector S. We use the associated Kendall's τ and its estimate $\hat{\tau}$ as a dependence measure.

$$D_j^{S,q}(\tau) = \frac{|\hat{\tau}_{j,I_k^{S,q}}|}{|\hat{\tau}_{j,I_k^{S,q}}| + |\hat{\tau}_{j,I_S^{S,q}|I_k^{S,q}}| + |\hat{\tau}_{j,o_{j_1}^{S,q}|I_k^{S,q},I_S^{S,q}}| + \dots + |\hat{\tau}_{j,o_l^{S,q}|I_k^{S,q},I^{S,q},o_{j_1}^{S,q},\dots,o_{j_{l-1}}^{S,q}}|}, \quad \forall_{S,q,j}. \tag{4}$$

It accounts for the conditional dependencies of the asset j to other assets $o_{j_1}^{S,q}, \dots, o_{j_{l-1}}^{S,q}$, where j_l is the number of assets which occur in the conditioning set, when j is in the conditioned set with the fitted vine in sector S and period q.

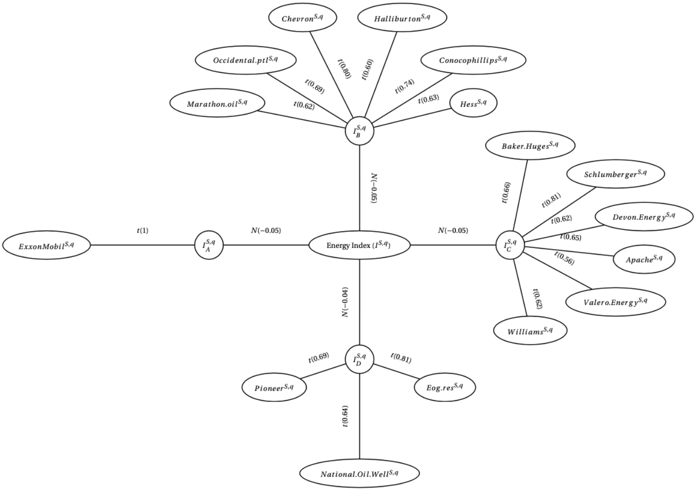
ESG class lower tail dependence indicator $D_j^{S,q}(\lambda)$

Similarly, we can also define the following ESG class lower tail dependence indicator $D_j^{S,q}(\lambda)$ for each asset j with its ESG class k within each sector S and period q.

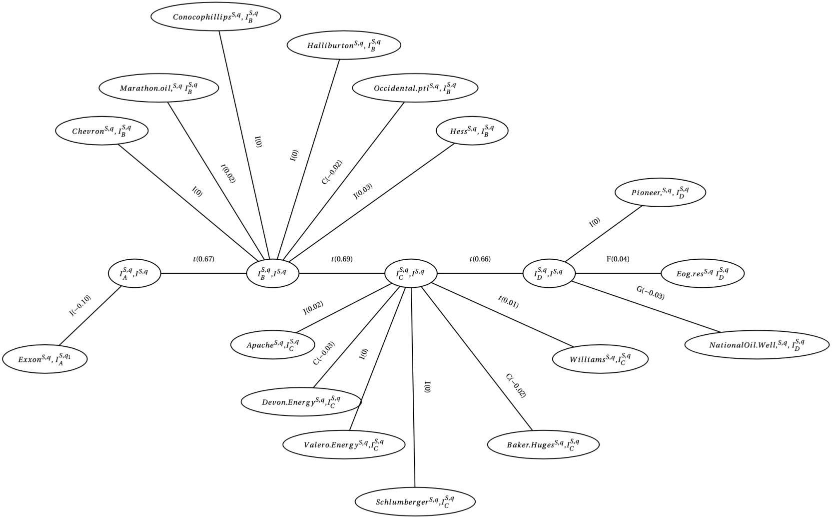
$$D_j^{S,q}(\lambda) = \frac{|\hat{\lambda}_{j,I_k^{S,q}}|}{|\hat{\lambda}_{j,I_k^{S,q}}| + |\hat{\lambda}_{j,I_S^{S,q}|I_k^{S,q}}| + |\hat{\lambda}_{j,o_{j1}^{S,q}|I_k^{S,q},I_S^{S,q}}| + \ldots + |\hat{\lambda}_{j,o_l^{S,q}|I_k^{S,q},I^{S,q},o_{j1}^{S,q},\ldots,o_{j_{l-1}}^{S,q}}|}, \quad \forall_{S,q,j}. \tag{5}$$

The lower tail dependence coefficient is also non-zero when the fitted bivariate copula class is Student's t, Clayton, 180° Joe, 180° Gumbel, BB1, BB7, or 180° BB8 in our model. For other bivariate copula families, we have zero lower tail dependence coefficient.Here λ is denotes the lower tail dependence coefficient, with estimate $\hat{\lambda}$.

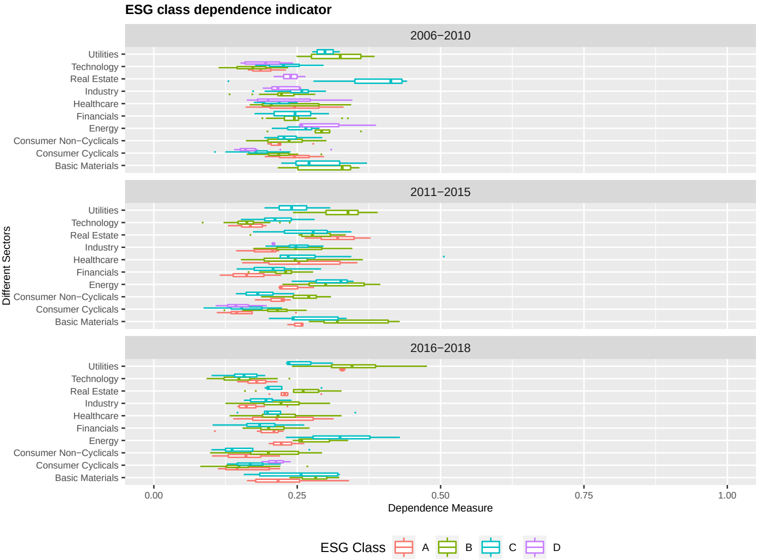
Energy Sector - Tree 1 - 2006-2010



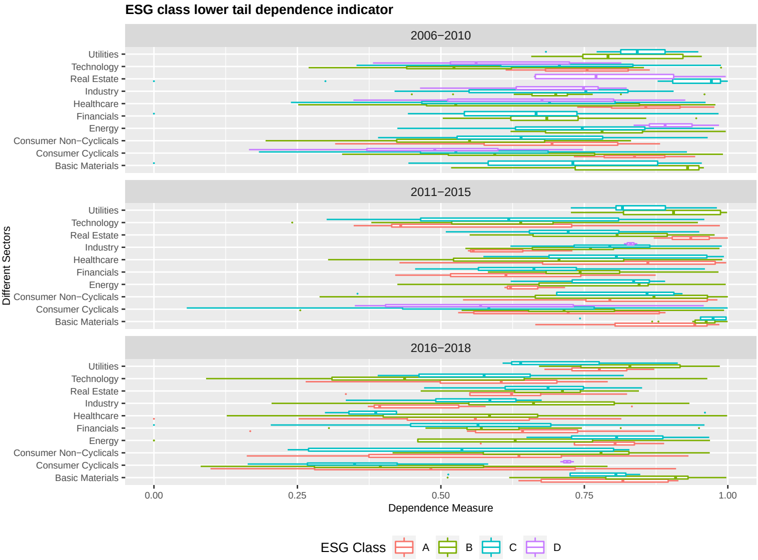
Energy Sector - Tree 2 - 2006-2010



ESG Class Dependence Indicator for each time interval



ESG Class Lower Dependence Indicator for each time interval



Mean ESG Class Dependence Indicator

Time Interval	2006-2010				2011-2015				2016-2018			
Sector | ESG class	*A*	*B*	*C*	*D*	*A*	*B*	*C*	*D*	*A*	*B*	*C*	*D*
Basic Materials	-	0.30	0.29	-	0.25	0.35	0.27	-	0.23	0.28	0.25	-
Consumer Cyclicals	0.25	0.22	0.18	0.18	0.16	0.21	0.15	0.15	0.16	0.15	0.17	0.21
Consumer Non-Cyclicals	0.22	0.23	0.23	-	0.22	0.26	0.19	-	0.16	0.20	0.16	-
Energy	-	0.29	0.25	0.30	0.24	0.31	0.30	-	0.23	0.28	0.33	-
Financials	-	0.25	0.24	-	0.17	0.23	0.21	-	0.19	0.21	0.18	-
Healthcare	0.25	0.23	0.22	0.24	0.26	0.24	0.28	-	0.22	0.22	0.22	-
Industrials	-	0.22	0.25	0.22	0.19	0.25	0.25	0.21	0.17	0.23	0.19	-
Real Estate	-	-	0.38	0.24	0.32	0.27	0.26	-	0.23	0.26	0.22	-
Technology	0.19	0.17	0.23	0.19	0.17	0.16	0.22	-	0.18	0.15	0.15	-
Utility	-	0.32	0.30	-	-	0.32	0.24	-	0.33	0.35	0.26	-

Table 3: $\bar{D}_k^{S,q}(\tau)$ for each Sector and ESG Class - Largest values in red

$$\bar{D}_k^{S,q}(\tau) = \frac{1}{n_k^{S,q}} \sum_{\substack{j' \in [1, n_S] \\ j' : R_{j'}^{S,q} = k}} D_{j'}^{S,q}(\tau), \quad \forall_{S,q,k}, \tag{6}$$

Mean ESG Class Lower Dependence Indicator

Time Interval	2006-2010				2011-2015				2016-2018			
Sector | ESG class	*A*	*B*	*C*	*D*	*A*	*B*	*C*	*D*	*A*	*B*	*C*	*D*
Basic Materials	-	0.82	0.68	-	0.86	0.95	0.93	-	0.79	0.75	0.74	-
Consumer Cyclicals	0.84	0.63	0.51	0.48	0.72	0.66	0.55	0.60	0.39	0.33	0.36	0.72
Consumer Non-Cyclicals	0.66	0.54	0.60	-	0.80	0.79	0.75	-	0.51	0.70	0.53	-
Energy	-	0.78	0.73	0.90	0.65	0.79	0.78	-	0.77	0.56	0.81	-
Financials	-	0.69	0.64	-	0.64	0.76	0.67	-	0.60	0.61	0.54	-
Healthcare	0.86	0.64	0.61	0.65	0.80	0.68	0.81	-	0.51	0.54	0.48	-
Industrials	-	0.69	0.69	0.69	0.61	0.76	0.81	0.83	0.49	0.64	0.56	-
Real Estate	-	-	0.85	0.80	0.94	0.78	0.74	-	0.40	0.69	0.67	-
Technology	0.75	0.55	0.61	0.60	0.58	0.59	0.63	-	0.58	0.46	0.57	-
Utility	-	0.81	0.84	-	-	0.89	0.85	-	0.78	0.83	0.72	-

Table 4: $\bar{D}_k^{S,q}(\lambda)$ for each Sector and ESG Class- Lowest values in blue

$$\bar{D}_k^{S,q}(\lambda) = \frac{1}{n_k^{S,q}} \sum_{\substack{j' \in [1, n_S] \\ j' : R_{j'}^{S,q} = k}} D_{j'}^{S,q}(\lambda), \quad \forall_{S,q,k}, \tag{7}$$

Conclusion

- By introducing an indicator to capture overall dependence among assets with the similar ESG scores and across sector, we are capable to quantify dependence.
- We see that such dependence is not negligible, with values often between 0.2 and 0.4, which tend to increase during crisis.
- Still, as the overall ESG dependence vary between 0.2 and 0.4, the idiosyncratic component for each stock as well as some other effects could still play a relevant role.
- We show that tail dependence tends to be higher during crisis.

Conclusion

- The understanding and estimation of such dependence is of utmost importance for setting up adequate risk management and mitigation tools as well as building portfolios, ideally also ESG diversified and resilient to crises.
- Current popular ESG inclusion approaches that focus on picking only assets in the highest ESG rating classes could have indeed possibly benefit in the past from better VaR values but such behavior is not clear for the most recent interval, where ESG classes are overlapping.

Conclusion

- In fact, picking assets with the highest ESG scores does not lead to better VaR values necessarily and could instead results in applying too much pressure on a specific set of assets, without a clear benefit.
- The constant trend in improving ESG scores might be a factor behind the lack of VaR differentiation between the classes A, B, and C in the last time interval, joint to the fact that such ESG scores are yet not definitive.
- Still, we notice that ESG class D assets tend to exhibit poorer VaR values than other ESG classes, suggesting that ESG disclosure might also have some indirect and positive effect on the company risk management.

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Data Notation

We introduce the mathematical indices, data sets and their notations used in the paper.

Type of Data	Notation
Sector	$S = 1, \ldots, 10$
Trading day	$t = 1, ..., 3271$
Year	$y = 2006, \ldots, 2018$
Period	$q = 1, 2, 3$
Total number of assets in sector S	n_S in Section Appendix B, Table B.7
Asset j in sector S	$j = 1, \ldots, n_S$
ESG class	$k \in \{A, B, C, D\}$
ESG score of asset j in sector S in year y	$ESG^S_{y,j}$
Log return of asset j in sector S in period q on trading day t belonging to ESG class k	$Y^{S,q}_{t,j,k}$
S&P 500 log return for sector S on trading day t in period q	$I^{S,q}_t$
Market capitalization weight of asset j in sector S (by 1.01.2015)	M^S_j
Mean ESG score of asset j in sector S and period q	$\overline{ESG}^{S,q}_j$
ESG class of asset j in sector S and period q	$R^{S,q}_j$
ESG class weight of asset j in sector S in period q	$\alpha^{S,q}_j$
Values of ESG class k in sector S and period q at trading day t	$\boldsymbol{I}^{S,q}_{t,k}$

$\overline{ESG}_j^{S,q}$

Mean ESG score of asset j in sector S and period q ($\overline{ESG}_j^{S,q}$)

$$\overline{ESG}_j^{S,q} = \frac{1}{|P_q|} \sum_{y \in P_q} ESG_{y,j}^{S} \quad \text{for} \quad \forall_{j,S,q}, \tag{8}$$

where $P_1 = [2006, 2010]$, $P_2 = [2011, 2015]$, $P_3 = [2016, 2018]$, and $|P_q|$ denotes the number of years in P_q.

$R_j^{S,q}$

ESG class of asset j in sector S and period q ($R_j^{S,q}$) We have $\forall_{j,S,q}$:

$$R_j^{S,q} = \begin{cases} A, & \text{if } \overline{ESG}_j^{S,q} \in [75, 100], \\ B, & \text{if } \overline{ESG}_j^{S,q} \in [50, 75), \\ C, & \text{if } \overline{ESG}_j^{S,q} \in [25, 50), \\ D, & \text{otherwise.} \end{cases} \tag{9}$$

$(\mathbf{I}_{t,k}^{S,q})$

Values of ESG class k in sector S and period q at trading day t $(\mathbf{I}_{t,k}^{S,q})$

$$\mathbf{I}_{t,k}^{S,q} = \sum_{\substack{j' \in [1, n_S] \\ j' : R_{j'}^{S,q} = k}} \alpha_{j'}^{S,q} \cdot Y_{t,j',k}^{S,q} \quad \text{for} \quad \forall_{S,q,k} \quad \text{and} \quad t \in T_q, \qquad (10)$$

where $T_1 = [1, 1260],\ T_2 = [1261, 2517],\ T_3[2518, 3271]$.

Two-Step Inference for Margins

As financial data are strongly dependent on past values and not uniformly distributed on $[0,1]^d$, which is the necessary input for a copula, a two-step inference for margins (IFM) approach is followed. This approach as been investigated by Joe (2005). We follow a parametric marginal model and estimate the margins first, we then use the estimated marginal distributions to transform the data on the copula scale by defining the pseudo-copula data. This allows us to remove the marginal time dependence by utilizing standard univariate time series models and then proceed with standardized residuals obtained from these models. We fit a generalized autoregressive conditional heteroskedasticity (GARCH) model with Student t innovations to our data, allowing for time-varying volatility and volatility clustering.

GARCH Model

Parameters	Notation
The set of trading days in period q	p_q with $p_0 = \emptyset$, $p_1 = \{1,\dots,1260\}$, $p_2 = \{1261,\dots,2517\}$, $p_3 = \{2518,\dots,3271\}$
S&P 500 log returns for sector S in period q	$\boldsymbol{I}^{S,q} = (I^S_{1+\lvert p_{q-1}\rvert},\dots,I^S_{\lvert p_{q-1}\rvert+\lvert p_q\rvert})^\top \in \mathbb{R}^{\lvert p_q\rvert}$
Matrix of log returns $Y^S_{t,j}$ in sector S and period q for $T \in t_q$	$Y^{S,q} = [\boldsymbol{Y}^S_1,\dots,\boldsymbol{Y}^S_{n_S}] \in \mathbb{R}^{\lvert p_q\rvert \times n_S}$, where $\boldsymbol{Y}^S_{j'} = (Y^S_{1+\lvert p_{q-1}\rvert,j'},\dots,Y^S_{\lvert p_{q-1}\rvert+\lvert p_q\rvert,j'})^\top$, $j' \in [1, n_S]$
Data matrix for sector S in period q	$X^{S,q} = [Y^{S,q},\boldsymbol{I}^{S,q},\boldsymbol{I}^{S,q}_A,\boldsymbol{I}^{S,q}_B,\boldsymbol{I}^{S,q}_C,\boldsymbol{I}^{S,q}_D] \in \mathbb{R}^{\lvert p_q\rvert \times P}$
Columns of the data matrix $X^{S,q}$	$d = 1,\dots,P$
Column d of the data matrix $X^{S,q}$	$\boldsymbol{X}^{S,q}_d = (X^{S,q}_{1,d},\dots,X^{S,q}_{\lvert p_q\rvert,d})^\top \in \mathbb{R}^{\lvert p_q\rvert}$
Conditional variance vector of $\boldsymbol{X}^{S,q}_{t,d}$ on trading day t	$(\sigma^{S,q}_{d,t})^2$
Estimated degree of freedom for $\boldsymbol{X}^{S,q}_{t,d}$	$\hat{\nu}^S_d$
Estimated distribution function of the innovation distribution for time series $\boldsymbol{X}^{S,q}_{t,d}$	$\hat{F}^{S,q}_d(\cdot\,;\hat{\nu}^S_d)$
Estimated u-data for an observation $X^{S,q}_{t,d}$ in sector S and period q	$\hat{u}^{S,q}_{t,d}$ where $t \in T_q$

GARCH Model continued

As an input of a R-vine model in sector S and period q, we have a data matrix $X^{S,q}$ defined in the Data Notation Section. Overall, we have $|S| \times |q| = 10 \times 3 = 30$ vine copula risk models.

In sector S and period q, we fit a GARCH(1,1) models with appropriate error distribution for a marginal time series, $\mathbf{X}_d^{S,q}$, and estimate the parameters of the following model:

$$\varepsilon_{d,t}^{S,q} = \sigma_{t,d}^{S,q} \cdot z_t \qquad (\sigma_{d,t}^{S,q})^2 = \gamma_0 + \gamma_1 \cdot (\varepsilon_{t-1,d}^{S,q})^2 + \beta_1 \cdot (\sigma_{t-1,d}^{S,q})^2 \tag{11}$$

where $(z_t)_{t>1}$ is a sequence of normal random independent and identically distributed random variables satisfying the standard assumptions $E[z_t] = 0$ and $var[z_t] = 1$ and follows a Student's t distribution.

GARCH Model continued

Then using the cumulative distribution function of the standardized Student's *t* distribution, we determine the *pseudo-copula data* as probability integral transformation (PIT), i.e.

$$\hat{u}_{t,d}^{S,q} := \hat{F}_d \left(\frac{X_{t,d}^{S,q}}{\hat{\sigma}_{t,d}^{S}}; \hat{\nu}_d^{S} \right). \tag{12}$$

Following this two-step approach allows us to convert data to the copula scale, which can be used for estimation of the copula parameter of the chosen bivariate copula family.

Number of Assets for each ESG class per Sector and interval

Time Interval	2006-2010				2011-2015				2016-2018			
Sector | ESG class	*A*	*B*	*C*	*D*	*A*	*B*	*C*	*D*	*A*	*B*	*C*	*D*
Basic Materials	1	7	11	0	3	11	5	0	5	10	4	0
Consumer Cyclicals	2	15	23	10	4	25	15	6	9	27	12	2
Consumer Non-Cyclicals	6	11	13	1	9	17	4	1	9	18	4	0
Energy	1	6	6	3	3	9	3	1	4	9	3	0
Financials	1	17	30	1	3	24	22	0	6	13	30	0
Healthcare	2	13	16	3	6	18	9	1	13	16	5	0
Industrials	1	15	20	5	3	22	14	2	7	24	10	0
Real Estate	1	1	14	4	2	7	10	1	6	10	4	0
Technology	6	16	21	8	7	25	18	1	10	32	9	0
Utilities	0	13	10	0	0	15	8	0	2	18	3	0

Table 5: Number of Assets for each ESG class per sector S and time period q

Two R-vine Copula ESG Risk Model

Two R-vine models for each sector S and time period q
$2 \cdot 10 \cdot 3 = 60$

The first R-vine model is fitted allowing only for the *itau* copula families and their rotations and reflections, while in the second R-vine model we allow for all *parametric* copula families and their rotations and reflections (all copula families are presented in Table 6).

	Itau[1] Copulas							BB Copulas		
Properties	*t*	*F*	*N*	*C*	*J*	*G*	**I**	*BB1*	*BB7*	*BB8*
Positive Dependence	✓	✓	✓	✓	✓	✓	-	✓	✓	✓
Negative Dependence	✓	✓	✓	-	-	-	-	-	-	-
Tail Asymmetry	-	-	-	✓	✓	✓	-	✓	✓	✓
Lower Tail Dependence	✓	-	-	✓	-	-	-	✓	✓	-
Upper Tail Dependence	✓	-	-	-	✓	✓	-	✓	✓	✓

Table 6: Parametric copula families and their properties without rotations and reflections. Notation of copula families:t = Student's t, F = Frank, N = Gaussian, C = Clayton, J = Joe, G = Gumbel, **I** = Independence, BB1 = Clayton- Gumbel, BB7 = Joe-Clayton, BB8= Extended Joe

[1]Copula families for which the parameter estimation by Kendall's τ inversion is available without rotations

Which model?

To choose our optimal model we use the Bayesian Information Criteria (BIC) as it tends to select a parsimonious model that reasonably approximates the density (Schwarz and Others, 1978).

Considering BIC values of the model fits indicate that *itau* bivariate copula family set is preferred in our data, except for the Real Estate sector in period $q_1 = 2006 - 2010$ and $q_2 = 2011 - 2015$. Therefore, we allowed for BB copula fits in these two cases. Since the number of parameters differ in the bivariate copula families that we considered, we used the parsimonious BIC correction in the Vuong test to favor more parsimonious models.

Fitted Copula Families - Itau

Sector	*BM.*[a]	*C. Cyc.*[b]	*N.Cyc.*[c]	*Ene.*[d]	*Fin.*[e]	*H.Care*[f]	*Ind.*[g]	*R. Est.*[h]	*Tech.*[i]	*Uti.*[j]
2006-2010	T_1	T_1	T_1	T_1	T_1	T_1	T_1	T_1	T_1	T_1
Student's t	18	44	30	16	50	33	44	20	50	23
Clayton	0	0	1	0	0	2	0	1	0	0
Frank	1	6	1	0	1	1	0	0	4	2
Gaussian	3	0	2	4	2	2	0	0	0	0
Gumbel	0	4	1	0	0	0	1	0	1	0
Independence	0	0	0	0	0	0	0	3	0	0
Joe	0	0	0	0	0	0	0	0	0	0
2011-2015	T_1	T_1	T_1	T_1	T_1	T_1	T_1	T_1	T_1	T_1
Student's t	19	43	31	15	50	34	41	20	50	23
Clayton	1	0	0	0	0	2	0	0	0	2
Frank	0	7	0	1	0	0	2	0	3	0
Gaussian	0	0	4	1	2	1	2	0	2	0
Gumbel	0	4	0	0	0	0	0	0	0	0
Independence	2	0	0	3	0	0	0	1	0	0
Joe	0	0	0	0	0	1	0	3	0	0
2016-2018	T_1	T_1	T_1	T_1	T_1	T_1	T_1	T_1	T_1	T_1
Student's t	18	41	29	15	50	33	41	18	50	23
Clayton	2	0	0	1	0	3	0	1	0	3
Frank	0	10	2	1	0	0	3	0	1	0
Gaussian	1	1	2	1	1	1	0	2	1	0
Gumbel	0	2	1	1	1	0	0	0	2	0
Independence	1	0	0	0	0	0	0	2	0	0
Joe	0	0	0	0	0	0	0	0	0	0

Table 7: Bivariate *itau* copula families and independence copula fitted for Tree 1 (T_1). To simplify, no difference in the counts are made based on rotations and reflections. These are available on request by the authors. ([a] Basic Materials, [b] Consumer Cyclicals, [c] Consumer Non-Cyclicals, [d] Energy, [e] Financials, [f] Healthcare, [g] Industrials, [h] Real Estate, [I] Technology, [J] Utility)

Energy Sector Model Fit

Comparison of the *itau* and *parametric* R-vine models.

In the following the BIC are compared from three different vine models for the Energy sector and time period. In every model the preselect feature of *rvinecopulib* is not activated. The output of all additional sectors and time intervals are available from the authors upon request. See Table 6 for copula family abbreviations.

	loglik	BIC
Itau 2006-2010	32393.89	-63074.45
Par 2006-2010	32363.61	-62899.67
Itau 2011-2015	30135.86	-58637.47
Par 2011-2015	29643.34	-57495.43
Itau 2016-2018	11526.09	-21727.11
Par. 2016-2018	11005.10	-20671.87

Table 8: Model Fit - Energy

Interval	Statistic	Schwarz Statistic	Schwarz p-value
2006-2010	2.012	5.806	6.383e-09
2011-2015	4.470	5.182	2.194e-07
2016-2018	6.210	6.289	3.190e-10

Table 9: Vuong Test - Energy